Mathematical Challenge July 2017

Asset Pricing Anomalies in Financial Markets: Low Risk Anomaly

References

- [1] H. Jacobs: “The limits of the market-wide limits of arbitrage: Insights from the dynamics of 100 anomalies”
- [2] M. Baker, B. Bradley and J. Wurgler, 2011: “Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Approach”
- [3] R. Haugen and A. J. Heins, 1975: „Risk and the Rate of Return on Financial Assets: Some Old Wine in New Bottles”
- [4] A. Frazzini, A. and L. H. Pedersen, 2013: “Betting Against Beta”

Description

Introduction

The efficient-market hypothesis developed by Fama states that asset prices fully reflect all available information. Therefore, according to the theory, no one can consistently outperform the market by using the same information that is already available to all investors.

Yet, a wide range of pricing anomalies are observed, suggesting that profitable portfolio strategies can be formulated [1]. Among the many candidates for the greatest anomaly, a particularly compelling one is the long-term success of low-volatility and low-beta portfolios [2], as indicated in Figure 1 below.

This challenge to the basic notion of a risk-return trade-off is not new. Haugen and Heins noted already in 1975 that there was little support for the notion that risk premiums manifest themselves in higher realized returns [3].

Of the many possible ways to create a low-beta portfolio with good risk-return characteristics, in this Mathematical Challenge we investigate the “Betting Against Beta” (“BAB”) strategy that creates a market-neutral portfolio using one-dimensional betas as laid out in [4].

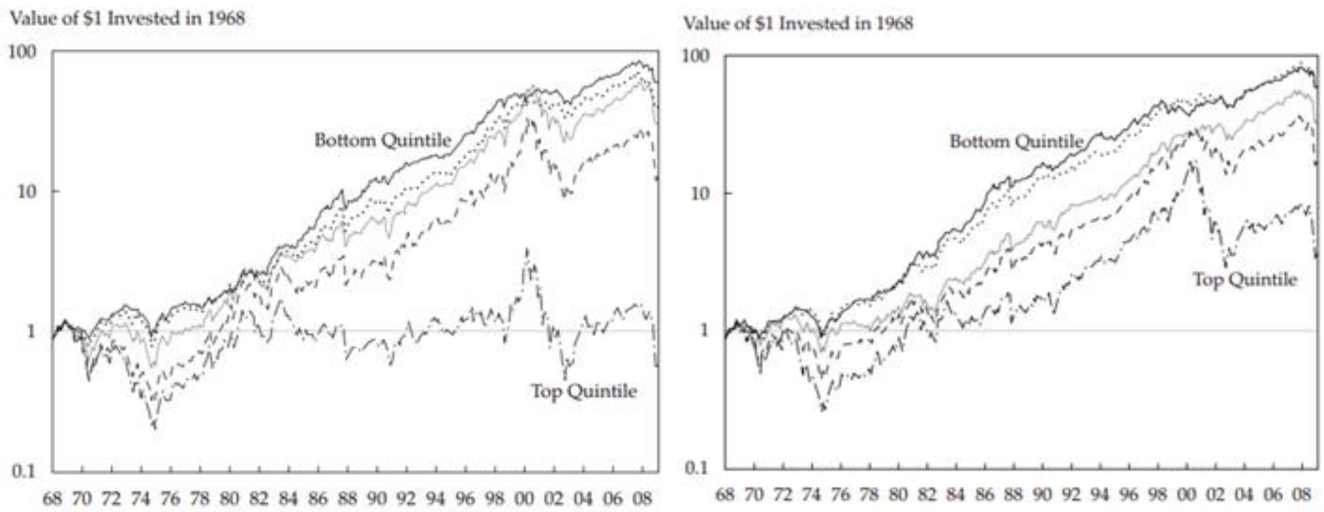

Figure 1: *Performance of US stocks since 1968, by volatility quintile (left) and beta quintile (right) [2].*

Formulation

The time-series of financial instruments r_i such as stocks can be regressed against market risk factors r_m, e.g. in the linear regression at time t

$$r_i = \alpha + \beta' r_m + \ \epsilon \ .$$

In [4], a self-financing market-neutral portfolio[1] is derived which takes leveraged long positions in low-beta instruments and finances these positions by short-selling high-beta instruments as follows:

Assume a list of n financial instruments $\left(r_{i,t}\right)_i$, *i=1,…,n*, and a market risk factor $r_{m,t}$, resulting in a list of betas $\left(\beta_{i,t}\right)_i$ for the instruments. Let $z_i = rank(\beta_{i,t})$ be the $n \ x \ 1$ vector of beta ranks at portfolio formation in descending order, and let $\bar{z} = 1_n' z/n$ be the average rank, where 1_n is a $n \ x \ 1$ vector of ones.

The idea now is to split the instruments into low-beta and high-beta categories, by defining the portfolio weights for the low-beta subportfolio w_L and the high-beta subportfolio w_H as

$$w_H = k(z - \bar{z})^+$$

$$w_L = k(z - \bar{z})^-,$$

where k is the normalizing constant $k = 2/1_n'|z - \bar{z}|$ and x^+ and x^- indicate the positive and negative elements of vector x. To construct final BAB portfolio and ensure that the portfolio is market-neutral, both subportfolios w_H and w_L have to be rescaled to have a beta of one at portfolio formation, e.g. at time 0 we require

$$\widetilde{w_H} = \ w_H'\left(\beta_{i,0}\right)_{i=1,\dots,n} \ = 1,$$

$$\widetilde{w_L} = \ w_L'\left(\beta_{i,0}\right)_{i=1,\dots,n} \ = 1.$$

Further details and additional background information can be found in [4].

[1] I.e., the portfolio beta is zero.

The objective is to modify the BAB methodology above to a multi-factor regression model where beta and r_m are multi-dimensional.

Technical details

Sharpe Ratio

One way to measure the performance of a portfolio is the Sharpe Ratio S_{pf}, which calculates the return relative to the volatility, and is defined as

$$S_{pf} = (r_{pf} - r_o)/\sigma_{pf},$$

where r_{pf} is the realized return of a portfolio and σ_{pf} is the realized volatility of a portfolio. r_o is the risk-free rate and is assumed to be zero for the following questions.

Questions:

- **Q1:** Implement the BAB strategy as laid out in [4] in a programming language of your choice. For the financial instruments, select instruments listed in the S&P 500 and use the S&P 500 as the market risk factor r_m for regression. Use historical data (e.g. from finance.yahoo.com) to derive betas and backtest the strategy using monthly rebalancing for the periods of 2003-2010 and 2010-2017. Compute the Sharpe Ratio for the strategy during these periods.
- **Q2:** The above methodology is only described for a one-factor model that is regressed against a single risk factor (i.e. resulting in one-dimensional beta). A one-factor model however is not enough to sufficiently explain the price fluctuations of instruments. Suggest a methodology extension to replicate the BAB strategy using a multi-dimensional beta.
- **Q3:** Implement your suggested methodology Q2 by selecting additional market risk factors for the instruments in Q1 and incorporate these into a multi-linear regression model. Backtest your strategy for the same periods as in Q1 and compute the Sharpe Ratio for your strategy.

We look forward to your opinions and insights.

Best Regards,

swissQuant Group Leadership Team