Mathematical Challenge October 2017

Replicating Portfolio Approach to Capital Calculation for Life Insurance Portfolios

References

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- [2] Natolski, J. and R. Werner (2016). Mathematical foundation of the replicating portfolio approach.
- [3] Pelsser, A. and J. Schweizer (2016). The difference between LSMC and replicating portfolio in insurance liability modeling. European Actuarial Journal 6(2), 441-494.
- [4] Cambou, M. and Filipović, D. (2016). Replicating Portfolio Approach to Capital Calculation. Finance and Stochastics, Forthcoming.
- [5] Li, M., Deng, S. J., and Zhoc, J. (2008). Closed-form approximations for spread option prices and Greeks. The Journal of Derivatives, 15(3), 58-80.

Description

Introduction

Both Solvency II and the Swiss Solvency Test require insurance companies to model the one-year P&L distribution of their asset-liability portfolio. The solvency capital is then determined as either the 99.5% VaR (Solvency II) or the 99% CVaR (Swiss Solvency Test) of this distribution.

The valuation of life insurance liabilities especially is very costly, requiring cash flow simulations up to 40 years and more. For this reason, insurance companies employ various proxy models (see [1]) that can be used to approximate the value of liabilities. This challenge is about one of the most popular proxy models, the Replicating Portfolio approach.

In practice, it is common to assume that the one-year P&L $PnL = PnL^{insurance} + PnL^{market}$ consists of two independent components, the insurance and the market P&L. We concentrate on PnL^{market} in this challenge.

Formulation

The goal of the replicating portfolio approach (see [2]) is to find a portfolio of financial instruments[1] $i = 1, \dots, N$ with cash flows A_t^i that replicate the value of a liability with cash flows L_t, $t = 1, \dots, T$, i.e.

[1]The financial instruments, such as bonds, interest rate swaps, swaptions and equity options, need not be tradeable. If they are, however, the replicating portfolio can also be used analyze hedging possibilities.

$$w_1^*, \dots, w_N^* = \underset{w_1,\dots,w_N}{\operatorname{argmin}}\, d\left(f, \sum_{i=1}^{N} w_i g_i\right)$$

Thereby, the optimal weights w_i^* depend on the distance function d, which is often chosen to be either the L1- or L2-norm and the target values f (of the liability) and g_i (of the instruments). These are usually either the discounted values, i.e., for discount factors D_t

$$f = \sum_{t=1}^{T} D_t L_t, \quad g_i = \sum_{t=1}^{T} D_t A_t^i$$

or vectors of cash flows[2], i.e.

$$f_t = L_t, \quad g_{t,i} = A_t^i.$$

Because there generally is no closed-form formula for the liability cash flows or values, the optimization needs to be scenario-based. In practice, replicating portfolios are not only calibrated against risk-neutral scenarios corresponding to the prevailing market conditions, but also stressed scenarios (e.g., yield curve shift of +100 bps). After the calibration, the risk measure (VaR or CVaR) is then calculated with respect to one-year-ahead real-world $\mathbb{P}$-scenarios.

While utilizing a big number of instruments helps to find a good solution of the optimization problem, it also leads to a possible overfit which could negatively impact out-of-sample performance (of which the $\mathbb{P}$ valuations are a special case). Therefore, practitioners need to find a balance between too many and too few instruments, and also consider the issue of potentially collinear instruments. To solve this problem, the optimization problem is often modified as follows

$$w_1^*, \dots, w_N^* = \underset{w_1,\dots,w_N}{\operatorname{argmin}}\, d\left(f, \sum_{i=1}^{N} w_i g_i\right) + \lambda \|w\|_p.$$

Thereby, the L1- and L2-norm are again commonly used for $\|w\|_p$, the former of which is called lasso regularization and the latter ridge regularization.

Questions

- Q1: What are the arguments for and against and the implications of
  - replicating discounted values versus replicating cash flows
  - minimizing the L1-norm versus minimizing the L2-norm
  - ridge regularization versus lasso regularization
- Q2: Compare the Replicating Portfolio approach to the Least-Squares Monte Carlo approach (see e.g., [2] or [3]). Show that a certain Replicating Portfolio formulation corresponds to the regress-later Least-Squares Monte Carlo approach. What are the basis functions?
- Q3: Consider the dynamic and path-dependent Replicating Portfolio approach in [4]. Assume that there are two financial instruments available for replication, which follow for $i = 1,2$

$$dS_t^i = S_t^i(r_i dt + \sigma_i dW_t^i), \qquad d\langle W^1, W^2\rangle_t = \rho dt.$$

[2] Note that if a portfolio of financial instruments perfectly replicates the cash flows of a liability, it follows by standard no-arbitrage arguments that the value of the portfolio must be equal to the value of the liability.

Further assume that the insurer is long one spread call option with maturity T and strike price K, i.e. has payoff

$$V_T = \max(0, S_T^1 - S_T^2 - K).$$

Compare the dynamic and path-dependent approach to the corresponding static approach in a numerical simulation using the closed-form approximation by [5].

We look forward to your opinions and insights.

Best regards,

swissQuant Group Leadership Team